Diffusivity of All-Pass Arrays

Vincent Daley
Thompson Rivers University

ABSTRACT

The diffuse properties of an all-pass loudspeaker line array were tested and compared to theoretically predicted results. The qualities and benefits of a diffuse loudspeaker array are first presented. Then, an exhaustive search method for determining optimal arrays is explored. It was found that computer optimized all-pass arrays can be easily constructed and prove to be diffuse sources as defined in this paper.

Introduction

Extensive literature exists on devices which promote diffuse sound fields in rooms. [1] Diffuse sound fields are desirable in acoustic settings because they provide uniform room coverage, improved boundary interaction, improved speech recognition, and a greater sense of envelopment by the sound. [2]

Traditionally, diffuse sound fields are achieved by surface treatments in a room, usually on the walls. [3] These treatments scatter and diffract sound in order to redistribute the sound energy in the room and reduce the impact of reflections off uniform surfaces. [3] Reflections off untreated surfaces cause unwanted flutters and echoes which are harmful to the aforementioned sound properties of room coverage, boundary interaction, speech intelligibility, and envelopment. [3]

Surface treatments are effective, but they are expensive, alter the appearance of a room and cannot easily be moved to a different space. For those reasons an apparatus which has diffusivity as a property of the direct sound field is more practical. In other words, a sound field which does not depend on boundary interactions to be diffuse is desired. This is known as a diffuse source. One such apparatus is the Distributed Mode Loudspeaker (DML). [2] DML's behave well and are considered diffuse by multiple sources [2],[4],[5] but they are not commonly used and the mechanism responsible for their diffusivity is poorly understood and simply said to be "complex". [2] Without the mechanism being properly understood the DML's behaviour cannot easily be optimized.

Another class of radiators which has some desirable properties in common with DML's, such as omnidirectionality and speech recognition, [6] is all-pass line arrays. An all-pass line array is a collection of wide frequency response loudspeakers. The loudspeakers are equally spaced along a straight line and supplied the same signal but with different polarity and amplitude at each element. Some examples of all-pass array driving functions include Bessel arrays [7],[8], quadratic phase arrays [8] and arrays based on optimized number sequences [6], among others. If an application is present in which either a DML or an all-pass array may be used the all-pass array is favourable because linear arrays are less expensive to construct, the response of linear arrays is well understood [6] and linear arrays are amenable to simulations which enable the tuning of array response to certain applications.

What is of interest currently is the diffusivity of all-pass linear arrays. Diffusivity can be quantified by determining the polar cross-correlation of the loudspeaker array. [1],[4] This is accomplished by measuring the acoustic signal in all directions around the loudspeaker. Correlation directivity plots measure the correlation at a fixed radius from the array between loudspeaker response at one angle against the response at a reference angle. [1],[4] Near the reference angle the correlation is expected to be high but if the source is diffuse it will drop off quickly as the angle away from reference increases. Low correlation off reference is indicative of a diffuse source. [4] Despite the promising acoustic theory assembled for all-arrays, the diffusivity of all-pass arrays had not yet been experimentally verified. [1] This paper will describe the computer optimization of an all-pass sequence, the construction of an all-pass loudspeaker array and the methods used to test the diffusivity of the array.

Theory

Consider a linear array of N sources with constant spacing "d," with elements that have differing strengths $A_0,\ldots, A_{N-1}$ but are otherwise identical. The array response at an angle θ in the far field can be represented by:

$$\hat{A}(\Omega) = \sum_{n=0}^{N-1} A_n e^{-in\Omega} \qquad (1)$$

TABLE 1. A few length-5 and 7 sequences with optimal decorrelation. The sequences were found by exhaustive search of sequences restricted to elements of value {0, ±0.1, ±0. 2, …, ±1}. [1] Note that all four sequences found to be optimal are skew-symmetric.

Sequence (A_n)	Ripple (dB)	C_w
(0.5, 1, 1, -1, 0.5)	1.3	0.53
(1, 1, 1, -1, 1)	5.1	0.45
(0.5, 1, 1, 0, -1, 1, -0.5)	1.0	0.47
(1, 1, 1, -0.4, -1, 1, -1)	5.7	0.40

as seen in [6]. Ω is related to θ via the relation $\Omega = \omega \frac{d}{c} sin\theta$, where ω is the frequency of the radiation and *c* is the speed of sound. It is useful to recognize the function $\hat{A}(\Omega)$ as the discrete time Fourier transform [9] of the sequence (A_N).

Equation (1) shows that the array response is completely determined by the function $\hat{A}(\Omega)$ and the Ω relation. At any given frequency (ω) the polar response (radiation pattern) is determined by $\hat{A}(\Omega)$ on the interval $[-\omega \frac{d}{c},\ \omega \frac{d}{c}]$ (the "visible region"). At low frequency (ω → 0) the visible region shrinks to a small neighborhood about **Ω**=0 and the array response tends to

$$\hat{A}(0) = \sum_{n=0}^{N-1} A_n \qquad (2)$$

which is independent of θ, so the radiation pattern becomes omnidirectional. With rising frequency, values of $\hat{A}(\Omega)$ on an increasingly wide interval map into the polar response. Consequently, the radiation pattern changes with frequency. Since $\hat{A}(\Omega)$ is a 2π-periodic function, for $\omega \frac{d}{c} > \pi \left(i.e. d > \frac{\lambda}{2}\right)$ this periodicity shows up in the array response $\hat{A}(\Omega)$ as periodic lobes (spatial aliasing) in the polar radiation pattern.

If the amplitude sequence (A_n) is chosen so that the magnitude spectrum

$$\left|\hat{A}(\Omega)\right| = G \qquad (3)$$

is constant, then from equation (1) we see that if each array element acts as an ideal point source with flat magnitude response and omnidirectional pattern then the array does as well. No finite-length sequence can satisfy (1) exactly. However, in practice we need only that (1) be satisfied to a good approximation. Finding sequences which do so is a well-known, long-standing and difficult problem in optimization. [10], [11] If the amplitudes A_n are restricted to a finite set of discrete values then an exhaustive search is an effective means of finding good sequences. [6] In sequence selection we use exhaustive search to find sequences that are optimized for both spectral flatness and diffuse radiation.

To quantify the correlation between signals that a source radiates in different directions, Gontcharov [2] introduced the polar cross-correlation plot as follows. Let $x_{\theta 1}(t)$ and $x_{\theta 2}(t)$ be the far-field signals radiated in directions θ_1 and θ_2, respectively. Their normalized cross-correlation function (CCF), defined by

$$c(\tau) = \frac{\int_{-\infty}^{\infty} x_{\theta_1}(t)x_{\theta_2}(t+\tau)dt}{\sqrt{\int_{-\infty}^{\infty} |x_{\theta_1}(t)|^2 dt \int_{-\infty}^{\infty} |x_{\theta_2}(t)|^2 dt}} \qquad (4)$$

gives the correlation between $x_{\theta 1}(t)$ and the time-shifted signal $x_{\theta 2}(t+\tau)$, taking values between +1 (perfect correlation) and −1 (perfect anti-correlation). As a single measure of the correlation between the two signals, we take the maximum of $|c(\tau)|$ over all values of τ to obtain the polar cross-correlation

$$C(\theta_1, \theta_2) = \max_{\tau} |c(\tau)|. \qquad (5)$$

For a given fixed reference angle θ_1, equation (5) gives a function of θ_2 only. In the balance of the paper we take the reference angle θ_1 = 0, perpendicular from the face of the array, and let θ_2 = θ vary, giving the polar cross-correlation

$$C(\theta) = \max_{\tau} |c(\tau)|. \qquad (6)$$

In [1] it was shown that for an all-pass array of ideal point sources the polar cross-correlation, C(θ), satisfies

$$C_w \equiv \frac{\max|A_n|}{\sqrt{\sum A_n^2}} \qquad (7)$$

when θ is much greater than zero. We also refer to the quantity C_w as the white noise cross-correlation.

Sequence Selection

Minimizing C_w is equivalent to maximizing the efficiency of array output. However, to design diffuse arrays with better spectral flatness, we need to find sequences (A_n) that simultaneously minimize both C_w and the spectral ripple, which we define as follows:

TABLE 2. Result of exhaustive search for optimal decorrelation of length 9, 11 and 13 sequences. The conditions imposed on the searches were: elements of value {0, ±0.1, ±0. 2, …, ±1}, skew-symmetric sequences and ripple ≤ 3.

Sequence (A_n)	Ripple (dB)	C_w
(0.6, -1, 1, -0.3, -1, 0.3, 1, 1, 0.6)	2.94	0.41
(-0.6, 0.6, -0.2, 0.6, -0.6, -0.6, 0.6, 0.6, 0.2, 0.6, 0.6)	2.90	0.33
(-0.8, 0.8, -0.3, 0.8, -0.8, -0.8, 0.8, 0.8, 0.3, 0.8, 0.8)	2.93	0.33
(0.7, -1, 0.9, -1, 1, 1, -1, -1, 1, 1, 0.9, 1, 0.7)	2.93	0.29
(1, -1, 0.9, -1, 0.7, 1, -1, -1, 0.7, 1, 0.9, 1, 1)	2.95	0.29

$$\text{spectral ripple} = \frac{\max_\Omega |\hat{A}(\Omega)|}{\min_\Omega |\hat{A}(\Omega)|}. \qquad (8)$$

Note that both C_w and the spectral ripple are invariant under the following sequence transformations:

1. sequence reversal;
2. multiplication of all the A_n by a non-zero scalar;
3. multiplication of each A_n by $(-1)^n$;

In particular, the last two transformations can be used to change any sequence to one where both A_0 and A_1 are non-negative. Thus, an exhaustive search need only consider this case, thus reducing by a factor of 4 the number of sequences to assess. Nevertheless, for longer sequences exhaustive search quickly becomes computationally infeasible.

Perusal of Table 1 indicates that many of the best odd-length sequences are skew-symmetric, i.e. with $n = 2m$ and

$$A_{m-i} = (-1)^i A_{m+i} \; (i = 1, \dots, m). \qquad (9)$$

A likely explanation is that skew-symmetric sequences have good auto-correlation properties by construction [11]. This suggests restricting the search to skew-symmetric sequences, which greatly shrinks the size of the search space, effectively doubling the sequence length for which an exhaustive search can be carried out for a given amount of computation.

To select the sequence, which would ultimately be implemented with loudspeakers and tested, an R script was written which searched for optimal sequences of a given length. The search accounted for the invariance conditions listed previously as well as the apparent superiority of skew-symmetric sequences. A few search results may be found in table 2. The sequence (A_n) = (-0.6,0.6,-0.2,0.6,-0.6, -0.6,0.6,0.6,0.2,0.6,0.6) was ultimately chosen for several reasons. Importantly, the ripple and white noise cross-correlation were low. Additionally, having only two different magnitude elements, with one being a scalar multiple of the other, simplifies the circuitry necessary to construct the array.

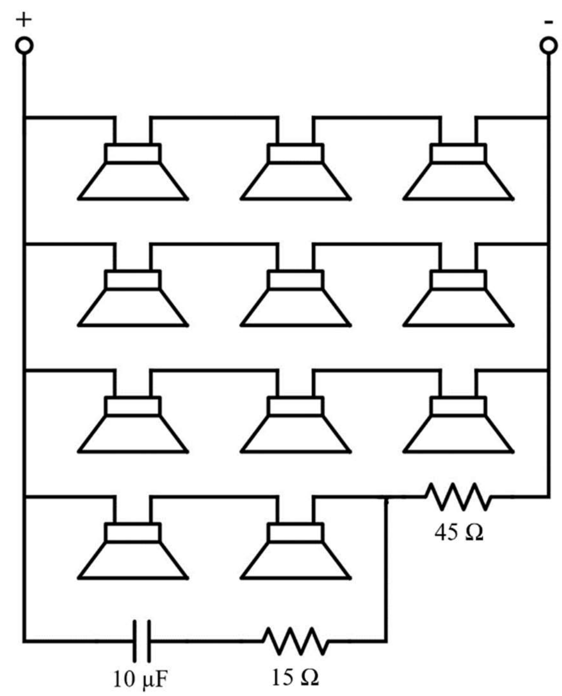

FIG. 1 Circuit used to implement desired optimal length 11 sequence. A combination of circuit analysis and trial and error testing was used to select components in the circuit. The capacitor-resistor portion is a Zobel network used to flatten the rising impedance response of the loudspeakers.

Construction and Testing Methods

Having chosen an appropriate sequence, the loudspeaker line array could then be assembled. The loudspeakers used were Dayton Audio ND65-8, chosen due to their availability and good range. Next, a circuit had to be designed which provided the necessary sequence (A_n) to the array elements.
A parallel combination with three elements per row proved to be ideal. The parallel configuration was ideal because it ensured the amplifier supplying the

signal “saw” the right amount of impedance. Dealing with negative elements was simple, the loudspeakers had to be wired backwards. (i.e. the input signal was provided to the negative terminal rather than the positive.) There were only two loudspeakers which had to have a reduced output compared to the others. That was accomplished by placing a resistor in series with the two lower amplitude loudspeakers. See figure 1 for a schematic of the circuit.

FIG 2. 11 element loudspeaker all-pass array. Each loudspeaker was backed by an insulated plastic tube and the box is made of plywood.

The larger resistor was chosen such that the two reduced amplitude elements used $\frac{1}{9}$ of the supply voltage (or $\frac{1}{3}$ the voltage of the other elements). The next consideration was the rising impedance of voice coil loudspeakers as frequency increases. With ND65-8 loudspeakers the effect becomes significant at frequencies greater than 1kHz. [12]

The rising impedance is an issue because it makes the loudspeaker voltages frequency dependent. Frequency dependence means that the desired levels in the array cannot be attained at all frequencies. This was not an issue for the rows which contained only loudspeakers, their impedances would rise uniformly and therefore they would all still use one third of the signal potential. The row with a resistor, however, would be affected because the carefully selected resistor would no longer correctly determine the voltage through the loudspeakers. The solution was to add an RC circuit in parallel with the two reduced amplitude loudspeakers. This is known as a Zobel network and it flattens the impedance curve of the loudspeakers making the two loudspeakers in the row frequency independent resistors. [13] The value of the capacitor in the Zobel network is specified by the following equation:

$$C = \frac{L}{R^2}. \tag{10}$$

Where L and R are the effective inductance and resistance values of the loudspeaker. The resistor used in the Zobel network is equal to the resistance of the loudspeaker.

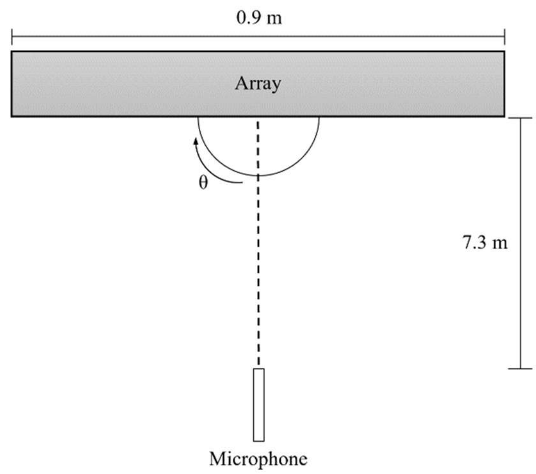

FIG 3. Configuration used to test the array. We were interested in the far field response, so the microphone was placed at a distance ~8 times the length of the array. The array was rotated through a 90-degree arc at 5-degree increment with a recording made at each angle.

Using Ohm’s law and equation (10) it was then possible to determine the values of the three external circuit components needed. Following analysis of the circuit and some trial and error refinements the component values used were: 45Ω for the resistor in series with the loudspeakers, 15Ω for the Zobel resistor and 10μF for the capacitor. Having designed a suitable circuit, the loudspeaker array could then be constructed. The speaker box was made of half inch plywood and the loudspeakers were mounted flush to the front, as seen in figure 2. 2-inch diameter and 11-inch long ABS pipe was also mounted to the back of each loudspeaker. The pipes were then stuffed with ~25g/L of pillow stuffing foam and capped on the far end. The stuffing was added in order to absorb as much of the sound radiating from the back of the elements as possible.

The testing procedure was quite straightforward. First a white noise signal low-pass filtered at 10kHz which would be supplied to the array was generated. The array was then placed outdoors on a flat but rough surface and a microphone was set-up 7.3m away. See figure 3 for a schematic of the test set-up. Both the output signal and the input from the microphone were managed by a computer, allowing for immediate analysis of the data collected.

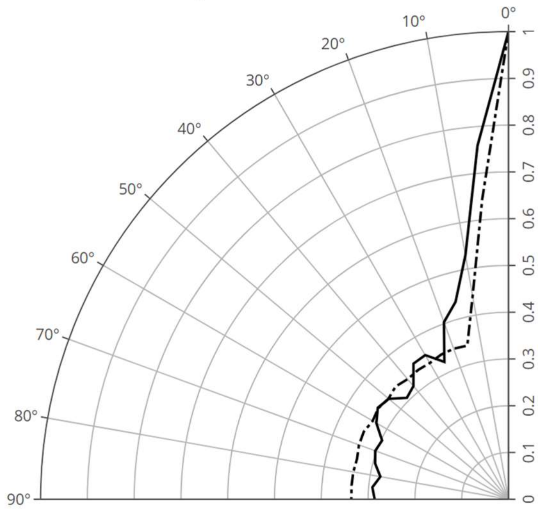

FIG 4. Measured polar cross-correlation plot for the array (A_n) = (-0.6,0.6,-0.2,0.6,-0.6, -0.6,0.6,0.6,0.2,0.6,0.6). The array spacing was 7.62cm and the feed signal was white noise low-pass filtered at 10kHz. The solid black line represents the experimental results found. The dashed line shows the expected behaviour of the same array.

Measurements were made with the microphone at various angles relative to the centre of the array. This was accomplished by rotating the array through 90^0 at 5^0 increments. At each position 3 trials were performed and the trial with the lowest noise was kept for subsequent analysis. The noise level was determined by taking the mean of each data set squared. The trial with the lowest mean value was said to have the least noise. Once all the measurements were made, it was possible to calculate the polar cross-correlation. The cross-correlation was calculated between a reference angle (0^0) and every other angle. After the polar cross-correlation was calculated it was plotted and compared to the results predicted by all-pass array theory.

Results

The results of the array testing are presented in figure 4. The dashed line illustrates the results expected according to equation (6). The solid line is the experimental result found. It was expected that the correlation would drop off to its minimum value once removed from the reference position by less than 15^0. However, we found correlation became minimal at greater than 20^0 from the reference position. The minimum measured correlation of 0.278 was lower than the theoretically expected 0.33. The other obvious discrepancy between theory and experiment is the wiggles in the experimental result. The array should have uniform cross-correlation once it reaches the minimum value, but the experiment found significant fluctuations.

Discussion

This research confirms the expected result; all-pass loudspeaker line arrays are diffuse sources as defined in this paper. The array tested was found to have polar cross-correlation (eq. (7)) similar both in magnitude and in polar to the theory in [1]. This means that the array constructed is a diffuse source and ought to possess the benefits of uniform room coverage, improved boundary interaction, improved speech recognition, and a sense of envelopment by the sound without the help of surface treatment. [2]

It is important to note, however, that none of those properties have been tested and compared to different acoustic sources in this work. Another limitation of this research comes in the refinement of the measurements. Ideally, acoustic measurements would have been made in an anechoic chamber. That was not possible at the time, so the measurements were made outdoors instead. That is as good an approximation to an anechoic space as was possible but there were limitations due to the lack of control over the testing environment. Other people and machines were making noise which interfered with the measurements. The other challenge with the location was limiting the interactions with surrounding surfaces. The array was placed as far from them as possible but there was a retaining wall ~5m from the array. Reflections off the wall were likely the major contributor to the fluctuations seen in the correlation of the array in figure 4.

It would have been interesting to build a second 11-element array in which each loudspeaker had the same amplitude and phase in order to get a baseline result and see both how that array was affected by the location, and how much the correlation is reduced by using an all-pass array. Additionally, more work could be done building and testing different sequence arrays to determine which behave best experimentally.

Conclusion

This research was undertaken to confirm the diffusivity of loudspeaker all-pass arrays. The findings agree with the existing theory. This work was important because no prior test of all-pass array diffusivity could be found in the literature explored. However, this being a first attempt also means that reproduction of the results and refinements to the method used must be made in order to increase the confidence in the results presented. Not only that but now that all-pass arrays have been shown to be diffuse, they may also be tested for the benefits which diffuse sources offer.

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